Kolmogorov-Smirnov Test for Discrimination of Dark Matter Velocity Distributions

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Directional detection for dark matter is expected to be capable of measuring its velocity distribution. We exploit a method to discriminate anisotropy of the distribution with ideal event number produced by Monte-Carlo simulation. In order to compare two energy-angular distributions, Kolmogorov-Smirnov test is studied.

1 Introduction

Directional dark matter detection is expected to be the way to measure the velocity distribution of dark matter. Most of the case the isotropic Maxwell-Boltzmann distribution is supposed as the velocity distribution, however, some N-body simulations and observations suggest the existence of anisotropic components in the distribution. Discrimination between the isotropic and anisotropic distributions in the directional detection with the chi squared test is discussed in previous study [1]. As well as the chi squared test, Kolmogorov-Smirnov test (KS test) [2] is a possible way to compare two distributions. In this study, the possibility to give constraints to the isotropic and anisotropic components is investigated with the KS test.

Following [1], to parametrize the anisotropic component in the velocity distribution we adopt anisotropy parameter r. It is defined as

$$f(v_\phi) = \frac{1-r}{N(v_{0,\text{iso.}})} \exp\left[-v_\phi^2/v_{0,\text{iso.}}^2\right] \quad (1)$$

$$+ \frac{r}{N(v_{0,\text{ani.}})} \exp\left[-(v_\phi-\mu)^2/v_{0,\text{ani.}}^2\right], \quad (2)$$

where v_ϕ is tangential velocity of dark matter in the galactic rest frame, the normalization factor $N(v_0) = 2v_0\Gamma(3/2)$, $v_{0,\text{iso.}} = 250$ km/s, $v_{0,\text{ani.}} = 120$ km/s and $\mu = 150$ km/s. Parameters associated with the anisotropic distribution are result of N-body simulations in reference [3]. Distributions for radial velocity and velocity across the galactic plane in the galactic rest frame are supposed to be Gaussian.

2 Kolmogorov-Smirnov test

If both recoil energy and scattering angle are detected by an experiment, we can make use of energy-angular distribution. In order to discriminate the distributions with different anisotropy parameter r, we adopt KS test. In the numerical calculation, KS test embedded in root package [4] is used. Two data sets are produced by Monte-Carlo simulation of elastic dark matter-target scattering: data of large event number $\sim O(10^8)$ and data with $O(10^{3-5})$ event number. The former is called as ideal "template data" and the latter is represented as "pseudo experimental data". For simplicity, mass relation $m_\chi = 3m_N$ is supposed in the simulation. No background signals and perfect resolutions are supposed in the analysis.

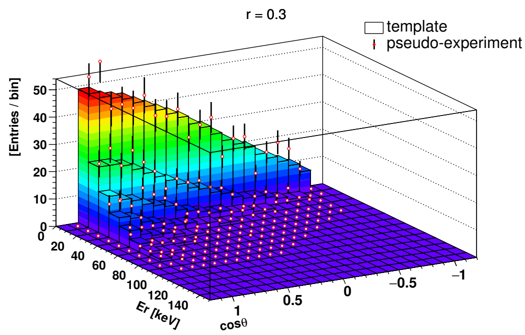

Figure 1 Energy-density distribution for scaled template data (3-dimensional bars) and pseudo experimental data (white dots).

An example of the energy-angular distribution is shown in Figure 1. As shown in the figure, the $E_R - \cos\theta$ plane is divided into small bins. We compare template data and pseudo experimental data by KS test.

In Figure 2–5, results of KS test for target fluorine (F) and silver (Ag) are shown. We produce one hundred data sets of pseudo experiments. Light green and yellow region in the figures represent 68% and 95% confidence intervals, respectively. Since tangential axises represent 1−(p-value),

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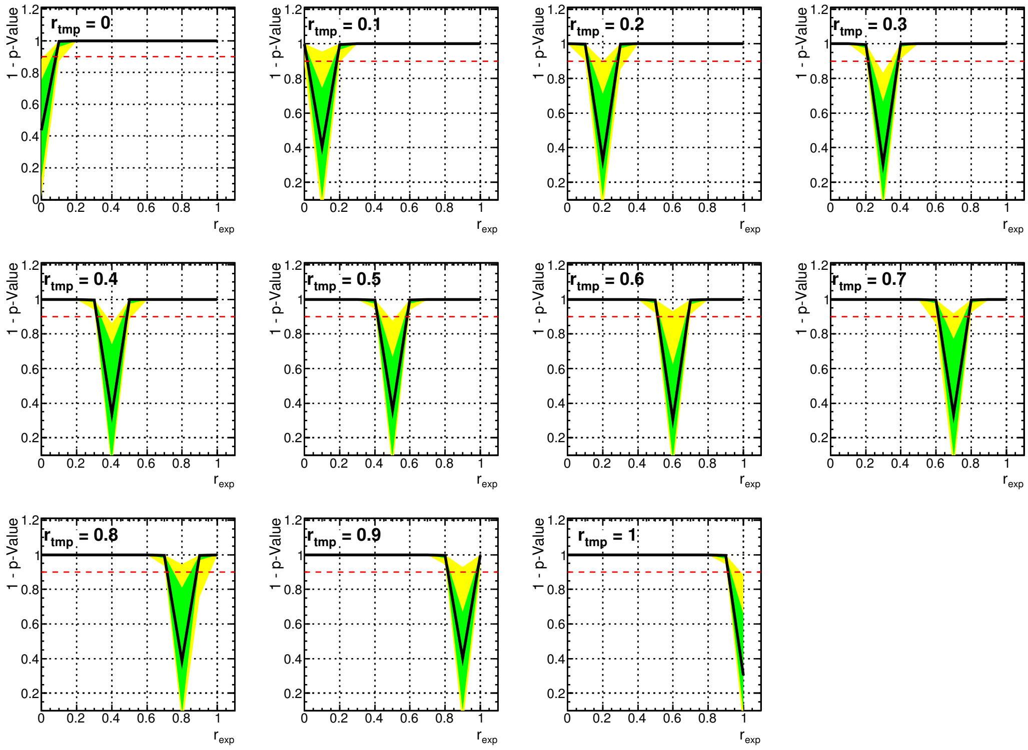

Figure 2　KS test between the anisotropy of template r_{template} and that for the pseudo-experiment $r_{\mathrm{exp.}}$. The target atom is F and the energy threshold $E_R^{\mathrm{thr.}} = 0$ keV. In the pseudo-experiment the event number 6×10^3. The red dashed line represents 90% CL.

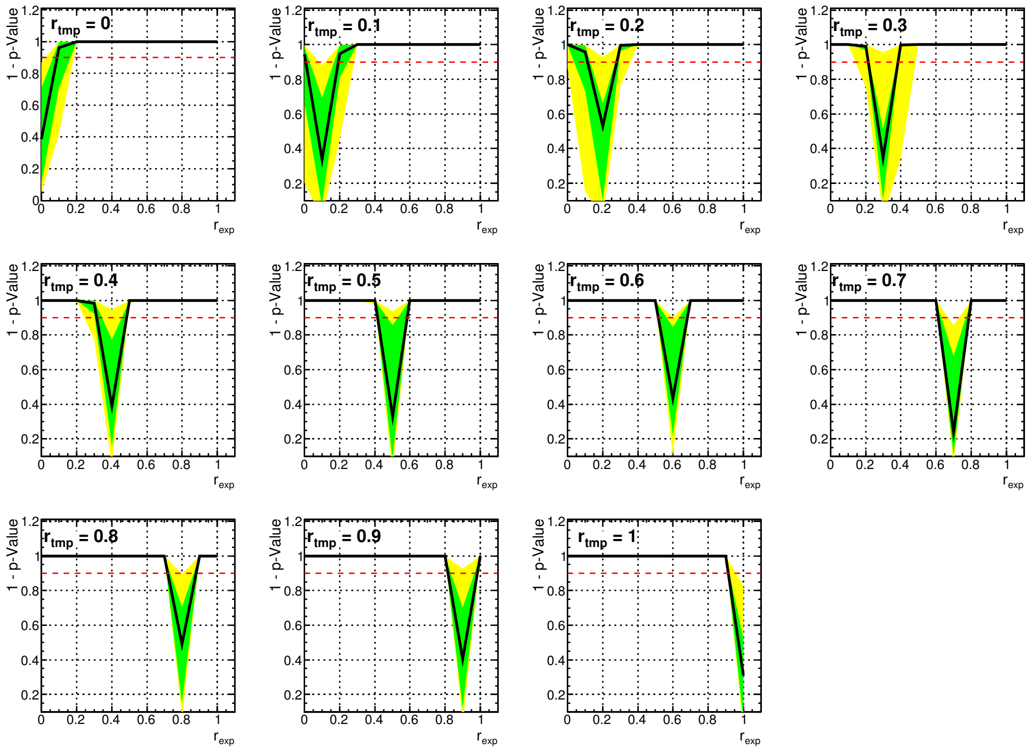

Figure 3　KS test between the anisotropy of template r_{template} and that for the pseudo-experiment $r_{\mathrm{exp.}}$. The target atom is F and the energy threshold $E_R^{\mathrm{thr.}} = 20$ keV. In the pseudo-experiment the event number 6×10^3. The red dashed line represents 90% CL.

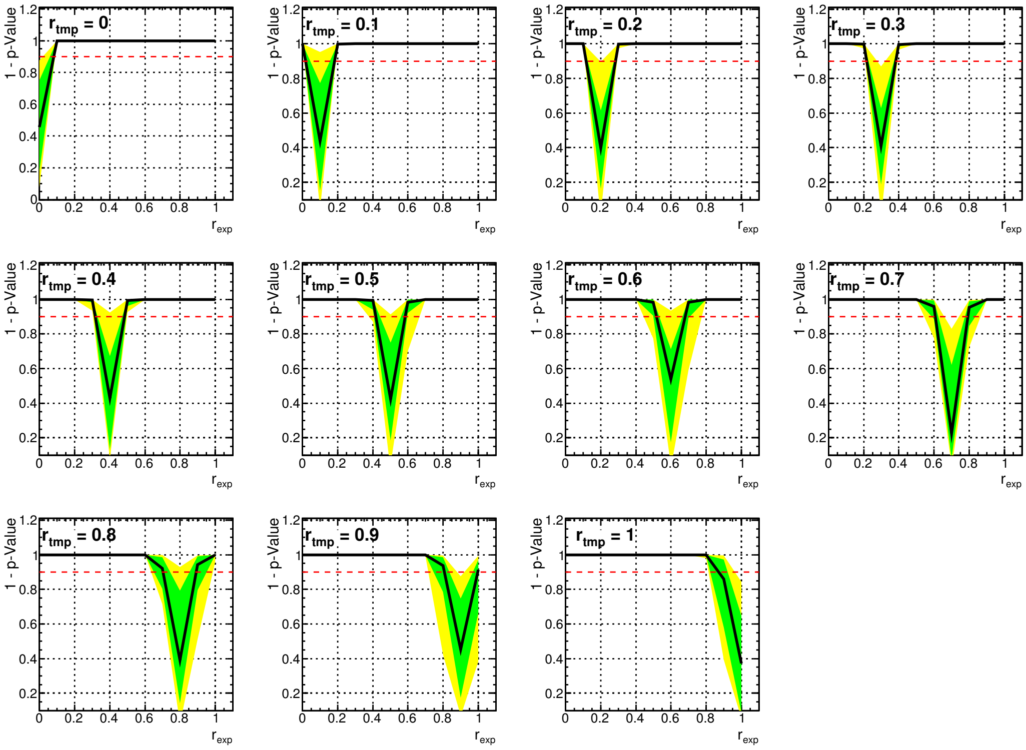

Figure 4 KS test between the anisotropy of template r_{template} and that for the pseudo-experiment $r_{\mathrm{exp.}}$. The target atom is Ag and the energy threshold $E_R^{\mathrm{thr.}} = 0$ keV. In the pseudo-experiment the event number 6×10^4. The red dashed line represents 90% CL.

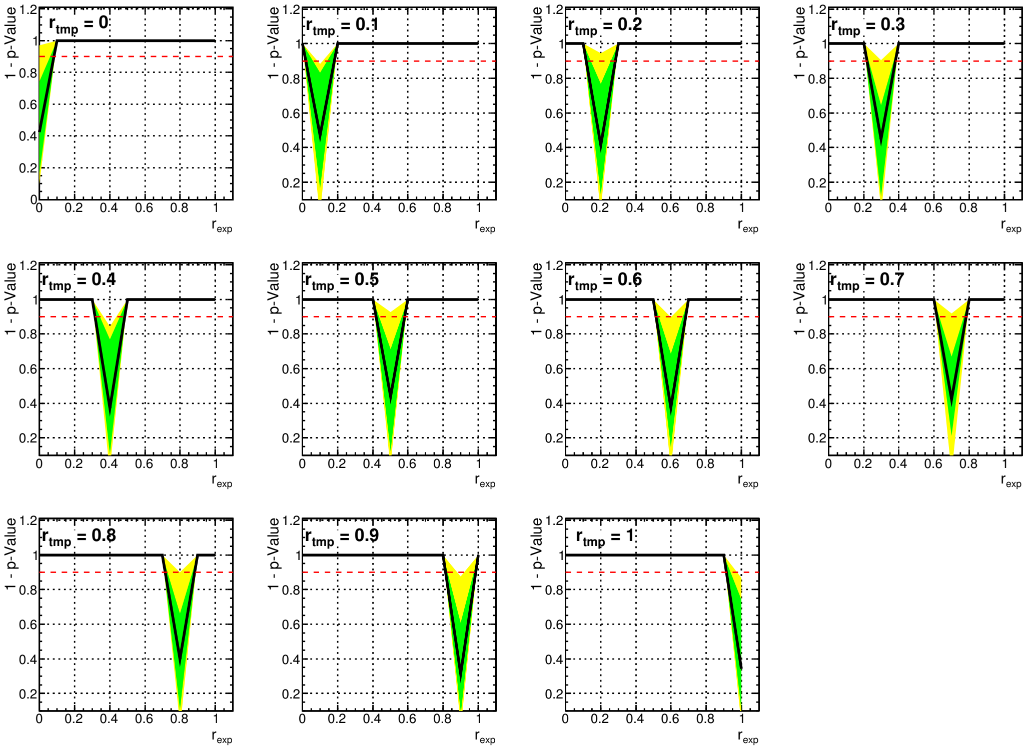

Figure 5 KS test between the anisotropy of template r_{template} and that for the pseudo-experiment $r_{\mathrm{exp.}}$. The target atom is Ag and the energy threshold $E_R^{\mathrm{thr.}} = 50$ keV. In the pseudo-experiment the event number 6×10^4. The red dashed line represents 90% CL.

upper regions from red dashed lines are excluded at 90% CL. F (Ag) is assumed as an typical light (heavy) target in the gaseous (solid) directional detector. The energy thresholds of 20 keV (F) and 50 keV (Ag) are assumed in Figure 3 and Figure 5, respectively, and the results for zero energy threshold are also shown for references in Figure 2 Figure 4. For target F and $r_{\mathrm{exp.}} = 0.2$, isotropic case $r_{\mathrm{exp.}} = 0$ can be excluded at 90% confidence level (CL.) with the pseudo-experimental data 6×10^3, while pseudo-experimental data 6×10^4 is required for target Ag. They are same order as chi squared test [1], however, statistic error is larger than case of chi squared test.

3 Conclusion

In this study, discrimination of anisotropy component in the velocity distribution of dark matter is analyzed with KS test. Required event number to exclude completely isotropic case at 90% CL. is $O(10^4)$ if anisotropy $r_{\mathrm{exp.}} = 0.2$ is realized. The order of event number is same as case of chi squared test. However, chi squared test is more suitable than KS test since the statistic error in case of KS test much larger than the error in case of chi squared test.

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